Hitchin 70
9-11 September 2016
University of Oxford

Abstracts of Talks

Bill Goldman (University of Maryland)

Title: The dynamics of classifying geometric structures

Abstract: The general theory of locally homogeneous geometric structures (flat Cartan connections) originated with Ehresmann's 1936 paper *Sur les espaces localement homogènes.* Their classification leads to interesting dynamical systems. For example, classifying Euclidean geometries on the torus leads to the usual action of the SL(2,Z) on the upper half-plane. This action is dynamically trivial, with a quotient space the familiar modular curve. In contrast, the classification of other simple geometries on the torus leads to the standard linear action of SL(2,Z) on R^2, with chaotic dynamics and a pathological quotient space. This talk describes such dynamical systems, where the moduli space is described by the nonlinear symmetries of cubic equations like Markoff's equation x^2 + y^2 + z^2 = x y z. Both trivial and chaotic dynamics arise simultaneously, relating to possibly singular hyperbolic metrics on surfaces of Euler characteristic equals -1.

Maxim Kontsevich (IHES)

Title: Noncommutative twistor families

Abstract: I will talk about a joint work with Y. Soibelman. We describe how a holomorphic symplectic manifold produces a twistor family of triangulated categories. The picture is based on the combination of ideas from Fukaya categories, from wall-crossing and from deformation quantization. In particular, we can extend the notion of harmonic metrics to q-difference equations.

Sasha Beilinson (University of Chicago)

Title: The singular support and the characteristic cycle

Abstract: I will describe a theory of ss & cc for 'etale sheaves on varieties over a field of any characteristic developed recently by Takeshi Saito and AB.

Marta Mazzocco (Loughborough University)

Title: Colliding holes in Riemann surfaces

Abstract: In 1997 Hitchin proved that the Riemann Hilbert correspondence between Fuchsian systems and conjugacy classes of representations of the fundamental group of the punctured sphere is a Poisson map. Since then, some generalisations of this result to the case of irregular singularities have been proposed by Boalch and by Gualtieri, Li and Pym. In this talk we interpret irregular singularities as the result of collisions of boundaries in a Riemann surface and show that the Stokes phenomenon corresponds to the presence of "bordered cusps". We introduce the concept of decorated character variety of a Riemann surface with bordered cusps and construct a generalised cluster algebra structure and cluster Poisson structure on it. We define the quantum cluster algebras of geometric type and show that they provide an explicit canonical quantisation of this Poisson structure.

Klaus Hulek (Leibniz Universität Hannover)

Title: Cubic hypersurfaces

Abstract: Cubic hypersurfaces are particularly interesting varieties which have played an important role in algebraic geometry. It is a classical result that every smooth cubic surface contains 27 lines. Cubic threefolds were shown by Clemens and Griffiths to be unirational but not rational. Very little is know about (non)-rationality of cubic fourfolds. Cubic threefolds and cubic fourfolds are also connected in multiple ways to abelian varieties and to irreducible holomorphic symplectic (hyperkähler) manifolds. In this talk I will describe some of these connections (old and new).

Shigefumi Mori (Kyoto University)

Title: Rational curves on algebraic varieties - minimal models and extremal rays

Abstract: I have been studying algebraic varieties through rational curves on them. I was first interested in a special problem called the Hartshorne Conjecture, and when I solved it I encountered a notion called an extremal ray as a biproduct, through which I got attracted to the biregular classification and the minimal model program, and furthermore to a general theory of higher dimensional birational classification. Reviewing them, I will also touch the study of 3-dimensional extremal contractions which I have been interested in.

Fedor Bogomolov (CIMS)

Title: Unramified correspondences and torsion of elliptic curves

Abstract: I will report on the results of an ongoing project which we began some years ago with Yuri Tschinkel and continue with Hang Fu and Jin Qian. We say that a smooth projective curve C dominates C' if there is a nonramified covering $\tilde C$ of C which has a surjection onto C'. Thanks to Bely's theorem we can show that any curve C' defined over $\bar{Q}$ is dominated by one of the curves $C_n, y^n-1= x^2$. Over $\bar F_p$ any curve in fact is dominated by C_6 which is in a way also a minimal possible curve with such a property. Conjecturally the same holds over $\bar Q$ but at the moment we can prove only partial results in this direction. There are not many methods to establish dominance for a particular pair of curves and the one we use is based on the study of torsion points and finite unramified covers of elliptic curves. In fact for any elliptic curve E over the complex numbers there is a uniquely defined subset of *4* points in P^1 modulo projective transformation defining the curve. These points correspond naturally to a subgroup of points of order *2* in E and there is a well defined (modulo projective transformation) subset of the images of torsion points from E in P^1. The corresponding subsets PE in P^1 for different elliptic curves E,E' have finite intersection and in many cases we can show that such an intersection consists of one point. On the other hand there are such subsets with intersection at least *22*. This raises a question about the existence of a universal upper bound for such intersections. This question is somewhat related to the question of Serre in the theory of Galois representations. On the any subset PE defines a bigger subset SE in P^1 by closing it by elliptic division. Namely SE contains PE and PE' for any E' defined by four points in SE. In the case when E is defined over $\bar{Q}$ we show that SE is projectively equivalent to $P^1(KE)$ where KE is an infinite extension of Q which is not equal to $\bar{Q}$ and varies for different elliptic curves over $\bar{Q}$.

Philip Candelas (University of Oxford)

Title: Zeta-functions for 1 parameter families of Calabi-Yau manifolds

Michael Atiyah (University of Edinburgh)

Title: The story of complex structures on the 6-sphere

Abstract: I will describe the genesis and history of this old problem, following it up to the present day.